

some proportionality coefficient.

Choosing the coordinate axes  $x_i$  coincident with the principal axes of the strain rate tensor, we obtain from (1.1) and the incompressibility condition

$$\varepsilon_{ii} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = u_{i,i} = 0 \tag{1.2}$$

a system of equations

$$[u_1]v_1 + [u_2]v_2 + [u_3]v_3 = 0 \tag{1.3}$$

$$[u_1]v_2 + [u_2]v_1 = [u_1]v_3 + [u_3]v_1 = [u_2]v_3 + [u_3]v_2 = 0 \tag{1.4}$$

Eqs. (1.4) have nontrivial solutions in  $[u_i]$  under the condition that  $v_1 v_2 v_3 = 0$ . To be definite, let us assume  $v_3 = 0$ , then it follows from (1.3), (1.4) that

$$v_1 = \pm 1/2 \sqrt{2}, v_2 = \pm 1/2 \sqrt{2}, [u_3] = 0 \tag{1.5}$$

Therefore, the discontinuous surfaces in the velocity coincide with the maximum shear surfaces (slip surfaces). The projection of the velocity vector on the principal direction in the tangent plane to the discontinuous surface is continuous upon passage through this surface.

Let us note that the slip surfaces for an arbitrary state of stress occur only under the Tresca plasticity conditions. For the rest of the plasticity conditions they are possible only for completely specific values of the stress deviator which will assure continuity of the stress deviator on the slip surface. From the continuity of the deviator and (0.4) we obtain that the stresses themselves are also continuous.

Slip surfaces along which the state of stress corresponds to the face of the Tresca plasticity condition are an exception. In this case the direction cosines of the principal axes of the stress tensor are continuous, and the mean principal stress tangent to the surface of discontinuity can undergo a discontinuity.

Let us turn to a determination of the relationship on the discontinuous surface  $\Sigma$ . The material experiences pure shear on the slip surface, hence, the strain rates are connected by the relationships [5]

$$\varepsilon_{ij} = \varepsilon_{ik} v_k v_j + \varepsilon_{jk} v_k v_i \tag{1.6}$$

The validity of (1.6) is seen easily by putting the coordinate axes coincident with the principal axes of the tensor  $\varepsilon_{ij}$ . We hence obtain

$$\varepsilon_i + \varepsilon_j = 0, \quad \varepsilon_k = 0, \quad v_i = \pm v_j = 1/2 \sqrt{2} \quad (i \neq j \neq k)$$

Let us introduce a curvilinear  $y_1, y_2$  coordinate system on the slip surface. Then the derivatives of the displacement velocities can be represented as

$$u_{i,j} = u_{i,n} v_j + g^{\alpha\beta} u_{i,\alpha} x_{j,\beta} \tag{1.7}$$

where  $g^{\alpha\beta}$  is the contravariant metric tensor of the surface,  $x_i(y_\alpha)$  is its parametric equation,  $u_{i,n}$  is the derivative of the vector  $u_i$  with respect to the normal  $n$  to the slip surface.

Substituting (1.7) into (0.3) we obtain

$$2\varepsilon_{ij} = u_{i,n} v_j + u_{j,n} v_i + g^{\alpha\beta} (u_{i,\alpha} x_{j,\beta} + u_{j,\alpha} x_{i,\beta}) \tag{1.8}$$

Utilizing (1.2) and (1.8), we obtain from (1.6) a system of three linear differential equations [5] in  $u_i$

$$u_{i,\sigma} x_{i,\tau} + u_{i,\tau} x_{i,\sigma} = 0 \tag{1.9}$$

Let us represent (1.9) somewhat differently

$$(u_i x_{i,\tau})_{,\sigma} + (u_i x_{i,\sigma})_{,\tau} - 2u_i x_{i,\tau\sigma} = 0 \tag{1.10}$$

Let us note that the relationships

$$x_{i, \tau\sigma} = \frac{\delta x_{i, \tau}}{\delta y_\sigma} + x_{i, \alpha} \Gamma_{\tau\sigma}^\alpha, \quad \frac{\delta x_{i, \tau}}{\delta y_\sigma} = b_{\tau\sigma} v_i, \quad u_i x_{i, \tau} = u_\tau \quad (1.11)$$

hold.

Here  $\delta x_{i, \tau} / \delta y_\sigma$  is the covariant derivative of the vector  $x_{i, \tau}$  with respect to  $y_\sigma$ ;  $\Gamma_{\tau\sigma}^\alpha$  is the Christoffel symbol corresponding to the metric of the surface,  $b_{\alpha\beta}$  are coefficients of the second fundamental quadratic form of the surface. Utilizing (1.11), we obtain from (1.10)

$$u_{\tau, \sigma} + u_{\sigma, \tau} - 2b_{\sigma\tau} u_n - 2\Gamma_{\sigma\tau}^\alpha u_\alpha = 0 \quad (1.12)$$

where  $u_n$  is the velocity vector component along the normal to the surface.

After passage from partial to covariant differentiation in (1.12), we obtain

$$\frac{\delta u_\tau}{\delta y_\sigma} + \frac{\delta u_\sigma}{\delta y_\tau} - 2b_{\sigma\tau} u_n = 0 \quad (1.13)$$

In the contravariant components of the velocity vector (1.13) becomes

$$g_{\sigma\alpha} \frac{\delta u^\alpha}{\delta y_\tau} + g_{\tau\alpha} \frac{\delta u^\alpha}{\delta y_\sigma} - 2b_{\sigma\tau} u_n = 0 \quad (1.14)$$

Multiplying (1.14) by  $g^{\sigma\tau}$ , we obtain after summation

$$2\Omega u_n = \delta u^\alpha / \delta y_\alpha \quad (1.15)$$

where  $2\Omega = b_{\alpha\beta} g^{\alpha\beta}$  is the mean curvature of the slip surface. Eliminating  $u_n$  from (1.14) by using (1.15), we have

$$\Omega \frac{\delta u^\alpha}{\delta y_\sigma} g_{\tau\alpha} + \Omega \frac{\delta u^\alpha}{\delta y_\tau} g_{\sigma\alpha} - b_{\sigma\tau} \frac{\delta u^\alpha}{\delta y_\alpha} = 0 \quad (1.16)$$

Only two of the three equations in (1.16) are independent since we obtain an identity after convolution with  $g^{\sigma\tau}$ . Therefore, the relationships (1.16) define two homogeneous equations with two unknown variables and two unknown functions. The asymptotic directions of the slip surface are characteristic for the system (1.16), and it is hyperbolic, elliptic, or parabolic if the Gaussian curvature of the surface is negative, positive, or zero, respectively. If the characteristic lines are chosen as curvilinear coordinates on the surface, then taking into account that  $b_{11} = b_{22} = 0$  ([7], p. 281), relationships (1.16) become  $g_{\tau\alpha} \delta u^\alpha / \delta y_\tau = 0$  (not summed over  $\tau$ ).

The relationships (1.12) hold on both sides of the velocity surface of discontinuity. Taking into account that the normal velocity component  $u_n$  is continuous, we obtain from (1.12)

$$[u_\tau]_{, \sigma} + [u_\sigma]_{, \tau} - 2\Gamma_{\sigma\tau}^\alpha [u_\alpha] = 0 \quad (1.17)$$

Let us select an orthogonal system of curvilinear coordinates  $y_\alpha$ , and let us direct the  $y_3$  axis along the principal axis in the tangent plane to the discontinuous surface. Taking into account that  $[u_2] = 0$ , we have from (1.17)

$$\Gamma_{22}^1 [u_1] = 0, \quad [u_1]_{, 2} - \Gamma_{12}^1 [u_1] = 0, \quad [u_1]_{, 1} - \Gamma_{11}^1 [u_1] = 0 \quad (1.18)$$

The Christoffel symbols in an orthogonal coordinate system are [7]:

$$\Gamma_{11}^1 = \frac{1}{2g_{11}} \frac{\partial g_{11}}{\partial y_1}, \quad \Gamma_{12}^1 = \frac{1}{2g_{11}} \frac{\partial g_{11}}{\partial y_2}, \quad \Gamma_{22}^1 = -\frac{1}{2g_{11}} \frac{\partial g_{22}}{\partial y_1} \quad (1.19)$$

Taking account of (1.19), we obtain from (1.18)

$$g_{22} = \varphi_1(y_2), \quad [u_1] = \varphi_2(y_1) \quad g_{11} = \varphi_3(y_2) \sqrt{g_{11}} \quad (1.20)$$

It follows from (1.20) that the geometry of the velocity surfaces of discontinuity should be such that the relationships

$$g_{22} = \varphi_1(y_2), \quad g_{11} = \{\varphi_3(y_2) / \varphi_2(y_1)\}^2 \quad (1.21)$$

will hold for  $g_{11}$  and  $g_{22}$  under the above-mentioned choice of curvilinear coordinates.

Let us make a change of coordinates on the discontinuous surface by means of Formulas

$$y_1 = F_1(z_1), \quad y_2 = F_2(z_2)$$

The lines  $y_\alpha = \text{const}$  hence go over into the lines  $z_\alpha = \text{const}$ . If  $F_1$  and  $F_2$  are selected so that  $dF_1/dz_1 = \varphi_1$ ,  $(dF_2/dz_2)^2 = 1/\varphi_1$ , then the metric tensor in the new coordinate system is  $\bar{g}_{22} = 1$ ,  $\bar{g}_{11} = \varphi_1^2 (F_2(z_2))$  (1.22)

It follows from (1.22) that the velocity surfaces of discontinuity can only be surfaces of rotation for any plasticity conditions except the edge of the Tresca plasticity condition, where one of the principal directions of the tensor  $\sigma_{ij}$  coincides with the directions of the geodesic lines.

In the plane strain case the surface  $\Sigma$  is a cylinder along whose generatrix  $u_2 = 0$ .

As the curves  $y_2$ ,  $y_1$ , respectively, let us take the generators of this surface and orthogonal lines, let  $y_\alpha$  be the distance along these lines measured from some point. The equalities

$$g_{\alpha\beta} = g^{\alpha\beta} = 1, \quad \Gamma_{\alpha\beta}^\sigma = 0, \quad 2\Omega = \kappa_1 = d\theta/dy_1, \quad u_\alpha = u^\alpha, \quad \delta u_\alpha/\delta y_\alpha = du_\alpha/dy_\alpha$$

hold for such a choice of the curvilinear coordinate system.

Here  $\kappa_1$  is the curvature of the line  $y_2 = \text{const}$ . Then taking account of these equalities, we obtain the Geiringer relationship from (1.15)

$$u_1 - u_n d\theta = 0$$

and from (1.18) we obtain that  $[u_1] = \text{const}$  on the slip surface.

If the state of stress corresponds to the edge of the Tresca flow surface, then

$$\sigma_1 = \sigma_2 = \sigma_3 \pm 2k \tag{1.23}$$

Hence, the direction of the third principal stress has been determined, and the directions of  $\sigma_1$  and  $\sigma_2$  remain undetermined. We represent the strain rates as

$$\epsilon_{ij} = \epsilon_1 l_i l_j + \epsilon_2 m_i m_j + \epsilon_3 n_i n_j \tag{1.24}$$

where  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$  are the principal values of the tensor  $\epsilon_{ij}$ ;  $l_i$ ,  $m_i$ ,  $n_i$  are the direction cosines of its principal axes which satisfy the conditions

$$l_i l_j + m_i m_j + n_i n_j = \delta_{ij} \tag{1.25}$$

As has been shown in [1 and 2], the system of equations for an ideal rigidly plastic body under the conditions (1.23) is statically determinate, i.e. the values of  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ ,  $n_i$  can be determined without involving (1.24). Hence, in examining the relationship (1.24) we will assume the  $n_i$  known, and the  $l_i$ ,  $m_i$  undetermined, but satisfying the conditions (1.25); where to satisfy the associated flow law it is necessary and sufficient to determine the tensor  $\epsilon_{ij}$  so that the relationships

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0, \quad |\epsilon_3| \geq |\epsilon_1|, \quad |\epsilon_3| \geq |\epsilon_2| \tag{1.26}$$

would hold.

Multiplying (1.24) by  $n_i$ , we obtain

$$\epsilon_{ij} n_j = \epsilon_3 n_i \tag{1.27}$$

Since  $\epsilon_3 = \epsilon_k j n_k n_j$ , the relationships (1.27) are written as

$$\epsilon_{ij} n_j = \epsilon_k j n_k n_j n_i \tag{1.28}$$

Only two of the Eqs. (1.28) are independent since the system (1.28) convoluted with  $n_i$  becomes an identity. We obtain the third independent equation from (1.24) by equa-

ting the subscripts  $i$  and  $j$ , and taking account of (1.26). We hence have  $\varepsilon_{ii} = 0$ .

Consequently, the kinematic relationships for the total plasticity condition can be written by using (0.3) as

$$u_{i,i} = 0, \quad u_{i,j}n_j + u_{j,i}n_j = 2u_{k,r}n_k n_r n_i \quad (1.29)$$

Let the total plasticity condition (1.23) hold on both sides of the velocity surface of discontinuity  $\Sigma$ . Because of the continuity of the stresses on this surface, the quantities are continuous, and we obtain from (1.29)

$$[u_{i,i}] = 0, \quad [u_{i,j}]n_j + [u_{j,i}]n_j = 2[u_{k,r}]n_k n_r n_i \quad (1.30)$$

It follows from the geometric relationships (1.7) that the jumps in the partial derivatives of the velocity on the discontinuous surface  $\Sigma$  are

$$[u_{i,j}] = B_i v_j + g^{\alpha\beta} [u_i]_{,\alpha} x_{j,\beta}, \quad B_i = [C_i] \quad (1.31)$$

Substituting their values from (1.31) in place of  $[u_{i,j}]$  in (1.30) we have

$$B_i v_i + g^{\alpha\beta} [u_i]_{,\alpha} x_{i,\beta} = 0 \quad (1.32)$$

$$\begin{aligned} (B_i v_j + B_j v_i) n_j + g^{\alpha\beta} ([u_i]_{,\alpha} x_{j,\beta} + [u_j]_{,\alpha} x_{i,\beta}) n_j = \\ = 2B_k n_k n_r v_r n_i + 2g^{\alpha\beta} [u_k]_{,\alpha} n_k x_{r,\beta} n_r n_i \end{aligned} \quad (1.33)$$

Eliminating the quantities  $B_i$  from (1.32), (1.33), and taking into account that  $n_i v_i = 1/2 \sqrt{2}$  on the velocity surface of discontinuity, we obtain

$$\sqrt{2} g^{\alpha\beta} [u_i]_{,\alpha} x_{j,\beta} n_j v_i - g^{\alpha\beta} [u_i]_{,\alpha} x_{i,\beta} = 2g^{\alpha\beta} [u_k]_{,\alpha} n_k x_{r,\beta} n_r \quad (1.34)$$

Since the projections of the velocity vector discontinuity on the principal direction in the tangent plane to the discontinuous surface equals zero, the vector  $[u_i]$  is in the same plane as the vectors  $n_i$  and  $v_i$ . Hence, taking account of the continuity of the normal component of the velocity vector to the surface  $\Sigma$ , we can represent  $[u_i]$  as

$$[u_i] = (\sqrt{2}n_i - v_i) V \quad (1.35)$$

where  $V(y_1, y_2)$  is a function characterizing the intensity of the velocity vector discontinuity.

Substituting its value from (1.35) in place of  $[u_i]$  in (1.34), we obtain

$$2\sqrt{2}g^{\alpha\beta} V_{,\alpha} n_i x_{i,\beta} + g^{\alpha\beta} (\sqrt{2}n_{i,\alpha} - v_{i,\alpha}) x_{i,\beta} V = 0 \quad (1.36)$$

Eq. (1.36) defines the connection of the intensity in the velocity discontinuity to the geometric characteristics of the surface  $\Sigma$ .

Since  $n_i v_i = 1/2 \sqrt{2}$ , the following holds:

$$g^{\alpha\beta} (\sqrt{2}n_{i,\alpha} - v_{i,\alpha}) x_{i,\beta} = \sqrt{2} g^{\alpha\beta} \{ (n_i x_{i,\beta})_{,\alpha} - \Gamma_{\alpha\beta}^{\sigma} x_{i,\sigma} n_i \} \quad (1.37)$$

Let us select orthogonal curvilinear coordinates such that  $y_1$  would coincide with the direction of the vector  $[u_j]$ . Then

$$n_i \frac{\partial x_i}{\partial y_1} = \left( \frac{g_{11}}{2} \right)^{1/2}, \quad n_i \frac{\partial x_i}{\partial y_2} = 0 \quad (1.38)$$

Taking account of (1.19), (1.38) and (1.37), let us transform (1.36) to

$$4g_{22} \frac{\partial V}{\partial y_1} + V \frac{\partial g_{22}}{\partial y_1} = 0 \quad (1.39)$$

After integrating (1.39), we obtain

$$(g_{22})^{1/4} V = (g_{22}^0)^{1/4} V_0 \quad (1.40)$$

where  $V_0$  and  $g_{22}^0$  are the values of  $V$  and  $g_{22}$  at some point on the line  $y_2 = \text{const.}$

Therefore, for a state of stress corresponding to the edge of the prism of the Tresca plasticity condition, the surfaces of velocity discontinuity can be of arbitrary shape, on which the integral (1.40) holds along a streamline of the vector  $[u_i]$ .

For any other plasticity condition, including even the Tresca faces, surfaces of velocity discontinuity are possible but they must be surfaces superposable on surfaces of revolution.

2. Let us consider the strain rate surface of discontinuity. On both sides of the surface  $G$  on which the displacement velocities are continuous but the strain rates undergo discontinuity, let the state of stress correspond to the smooth section of the plasticity condition. According to the results in [8], the strain rates are continuous on the surface of stress discontinuity. An exception is a face of the Tresca prism.

In this case the strain rates may undergo a discontinuity on the stress discontinuous surface; however, the direction cosines of the principal axes of the stress tensor will be continuous. Hence, it follows that the stresses are continuous on the strain rate discontinuous surface, with the possible exception of the Tresca face.

We consequently obtain from (0.3)

$$[e_{ij}] = [\lambda]f_{ij} \tag{2.1}$$

The system of equations (2.1) holds even for the face of the Tresca plasticity condition since in this case the quantity  $f_{ij}$  depends only on the direction cosines of the principal axes of the stress tensor, which are continuous. From (0.3) and (2.1) we have

$$[e_{ij}] = \frac{1}{2} ([u_{i,j}] + [u_{j,i}]) = [\lambda]f_{ij} \tag{2.2}$$

Since the displacement velocities are continuous, then  $[u_{i,j}] = B_i v_j$ , and (2.2) becomes

$$B_i v_j + B_j v_i = 2[\lambda]f_{ij} \tag{2.3}$$

Selecting the coordinate axes to coincide with the principal axes of the stress tensor, we obtain from (2.3) and the incompressibility condition (1.2)

$$B_1 v_1 + B_2 v_2 + B_3 v_3 = 0 \tag{2.4}$$

$$B_1 v_2 + B_2 v_1 = B_1 v_3 + B_3 v_1 = B_2 v_3 + B_3 v_2 = 0 \tag{2.5}$$

Comparing (2.4), (2.5) with (1.3), (1.4), we note that they will agree if the quantity  $[u_i]$  is substituted instead of  $B_i$  in (2.4), (2.5). Hence, all the properties obtained for the discontinuous surfaces of the displacement velocities will also hold for the discontinuous surfaces of the strain rates. In particular, the strain rate discontinuous surfaces coincide with the surfaces of maximum shear, one of the principal directions of the tensor  $\sigma_{ij}$  and the vector  $B_i$  lie in the tangent plane to the surface  $G$  and are mutually orthogonal.

Let us find differential relationships for the strain rate discontinuities along the surface  $G$ . Let the state of stress satisfy the Tresca face. Then the relationships of the associated flow law (2.2) can be represented in the form

$$e_{ij} = \lambda (m_i m_j - n_i n_j) \tag{2.6}$$

Let us note that the quantities  $m_i, n_i, l_i$  are continuous on the surface  $G$  and satisfy the conditions

$$n_i n_i = m_i m_i = l_i l_i = 1, \quad n_i m_i = n_i l_i = l_i m_i = 0 \tag{2.7}$$

Let us introduce into the considerations

$$M_i = \frac{dm_i}{dn}, \quad N_i = \frac{dn_i}{dn}, \quad L_i = \frac{dl_i}{dn}$$

It then follows from (2.7) that

$$\begin{aligned} M_i m_i = N_i n_i = L_i l_i = 0, & \quad M_i l_i + L_i m_i = 0 \\ N_i l_i + L_i n_i = 0, & \quad N_i m_i + M_i n_i = 0 \end{aligned}$$

i.e. the vectors  $N_i, M_i, L_i$  are orthogonal to the vectors  $n_i, m_i, l_i$ , respectively, and are representable as

$$N_i = -Nm_i + Ll_i, \quad M_i = Nn_i + Ml_i, \quad L_i = -Mm_i - Ln_i \quad (2.8)$$

It follows from (2.6) that

$$\begin{aligned} [e_{ij, k}] v_k = B(m_i m_j - n_i n_j) + [\lambda] (m_i \langle M_j \rangle + \langle M_i \rangle m_j - n_i \langle N_j \rangle - \\ - \langle N_i \rangle n_j) + \langle \lambda \rangle (m_i [M_j] + [M_i] m_j - n_i [N_j] - [N_i] n_j), \quad B = [d\lambda/dn] \quad (2.9) \end{aligned}$$

Here the symbol  $\langle \dots \rangle$  denotes the mean value of the appropriate quantities; the identity  $[ab] = \langle a \rangle [b] + [a] \langle b \rangle$  is used in the derivation. Using the second order geometric compatibility conditions [3]

$$[u_{i, jk}] = A_i v_j v_k + g^{\alpha\beta} B_{i, \alpha} (x_{j, \beta} v_k + x_{k, \beta} v_j) - B_i g^{\alpha\beta} g^{\sigma\tau} b_{\alpha\sigma} x_{j, \beta} x_{k, \tau}$$

$(A_i = d^2 u_i / dn^2)$

and taking account of (2.8), we obtain from (2.9)

$$\begin{aligned} A_i v_j + A_j v_i + g^{\alpha\beta} (B_{i, \alpha} x_{j, \beta} + B_{j, \alpha} x_{i, \beta}) = 2 \{ B(m_i m_j - n_i n_j) + \\ + P(m_i n_j + n_i m_j) + Q(m_i l_j + m_j l_i) - R(n_i l_j + l_i n_j) \} \quad (2.10) \end{aligned}$$

$$P = 2([N] \langle \lambda \rangle + \langle N \rangle \langle [\lambda] \rangle), \quad Q = [\lambda] \langle M \rangle + [M] \langle \lambda \rangle, \quad R = [\lambda] \langle L \rangle + [L] \langle \lambda \rangle$$

Equating the subscripts  $i$  and  $j$  in (2.10), we have

$$A_i v_i + g^{\alpha\beta} B_{i, \alpha} x_{i, \beta} = 0 \quad (2.11)$$

After multiplying (2.10) by  $v_j$  and taking account of (2.11), we obtain

$$A_i = g^{\alpha\beta} B_{k, \alpha} (x_{k, \beta} v_i - x_{i, \beta} v_k) + \sqrt{2} \{ B(m_i - n_i) + P(m_i + n_i) + (Q - R) l_i \}$$

It has here been taken into account that on the surface  $G$

$$n_i v_i = m_i v_i = \frac{1}{2} \sqrt{2}, \quad l_i v_i = 0$$

Let us note that the vector  $v_i$  can be represented as

$$v_i = \frac{1}{2} \sqrt{2} (m_i + n_i) \quad (2.13)$$

Eliminating the  $A_i$  from (2.10) by utilizing (2.12), and taking account of (2.13), we obtain

$$\begin{aligned} g^{\alpha\beta} B_{k, \alpha} \{ 2x_{k, \beta} v_i v_j - (x_{i, \beta} v_j + x_{j, \beta} v_i) v_k \} + g^{\alpha\beta} (x_{i, \alpha} B_{j, \beta} + x_{j, \alpha} B_{i, \beta}) = \\ = -2P(m_i m_j + n_i n_j) + (Q + R)(m_i l_j + l_i m_j - l_i n_j - l_j n_i) \quad (2.14) \end{aligned}$$

Only three of the six Eqs. (2.14) are independent, since they reduce to one equation after the subscripts  $i$  and  $j$  have been equalized, or after having been multiplied by  $v_i, v_j$ .

To determine the independent equations, let us multiply (2.14) by  $x_{i, \sigma} x_{j, \tau}$  whence we have

$$\begin{aligned} B_{i, \tau} x_{i, \sigma} + B_{i, \sigma} x_{i, \tau} = -2P(m_i m_j + n_i n_j) x_{i, \tau} x_{j, \sigma} + \\ + (Q + R)(m_i l_j + m_j l_i - n_i l_j - n_j l_i) x_{i, \tau} x_{j, \sigma} \quad (2.15) \end{aligned}$$

Transforming the left side of (2.15) by the scheme for the transformation of (1.9), we obtain

$$B_{\tau, \sigma} + B_{\sigma, \tau} - 2\Gamma_{\tau\sigma}^{\alpha} B_{\alpha} = -2P(m_i m_j + n_i n_j) x_{i, \tau} x_{j, \sigma} + (Q + R)(m_i l_j + l_i m_j - n_i l_j - n_j l_i) x_{i, \tau} x_{j, \sigma} \quad (2.16)$$

Let us represent the vectors  $x_{i, \tau}$  as

$$\frac{\partial x_i}{\partial y_1} = \frac{\sqrt{2}}{2} \sqrt{g_{11}} (m_i - n_i), \quad \frac{\partial x_i}{\partial y_2} = \sqrt{g_{22}} l_i \quad (2.17)$$

The curvilinear mesh  $y_1, y_2$  has here been chosen orthogonal, hence the directions of  $B_i$  and  $y_1$  coincide.

Substituting (2.17) into (2.16), we obtain

$$\frac{\partial B_1}{\partial y_1} - \Gamma_{11}^1 B_1 = -P g_{11}, \quad \Gamma_{22}^1 = 0, \quad \frac{\partial B_1}{\partial y_2} = 2\Gamma_{12}^1 B_1 + (Q + R)(2g_{11} g_{22})^{1/2} \quad (2.18)$$

From the second equation of (2.18) and (1.19) we obtain that

$$g_{22} = g_{22}(y_2)$$

It hence follows that the intermediate principal axes of the tensor  $\sigma_{ij}$  coincide with the directions of the geodesic lines on the strain rate surface of discontinuity.

#### BIBLIOGRAPHY

1. Ivlev, D. D., On the general equations of the theory of ideal plasticity and of statics of granular media, PMM Vol. 22, №1, 1958.
2. Ivlev, D. D., On relationships governing plastic flow under the Tresca plasticity condition and its generalization, Dokl. Akad. Nauk SSSR, Vol. 124, №3, 1959.
3. Thomas, T., Plastic flow and fracture in solids, New York, London, Acad. Press, 1961.
4. Bykovtsev, G. I., Ivlev, D. D. and Martynova, T. N., On properties of the general equations of the theory of an isotropic ideal plastic body with piecewise-linear potentials, Izv. Akad. Nauk SSSR, Mekhanika, №1, 1965.
5. Bykovtsev, G. I. and Miasniankin, Iu. M., On slip surfaces in three-dimensional rigidly plastic bodies, Dokl. Akad. Nauk SSSR, Vol. 167, №6, 1966.
6. Hill, R., Discontinuity relations in mechanics of solids, In: Progress in Solid Mechanics, Vol. 2, 1961.
7. McConnell, A. J., Application of Tensor Analysis, New York, Dover Publ. 1957.
8. Bykovtsev, G. I., Ivlev, D. D. and Miasniankin, Iu. M., On relationships on the stress discontinuous surfaces in three-dimensional ideal rigidly plastic bodies, Dokl. Akad. Nauk SSSR, Vol. 177, №5, 1967.

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